10.1: 17-22, 34, 38

17. $x = \cos 4t$, $y = t$, $z = \sin 4t$. At any point (x, y, z) on the curve, $x^2 + z^2 = \cos^2 4t + \sin^2 4t = 1$. So the curve lies on a circular cylinder with axis the y-axis. Since $y = t$, this is a helix. So the graph is VI.

18. $x = t$, $y = t^2$, $z = e^{-t}$. At any point on the curve, $y = x^2$. So the curve lies on the parabolic cylinder $y = x^2$. Note that y and z are positive for all t, and the point $(0, 0, 1)$ is on the curve (when $t = 0$). As $t \to \infty$, $(x, y, z) \to (\infty, \infty, 0)$, while as $t \to -\infty$, $(x, y, z) \to (-\infty, \infty, \infty)$, so the graph must be II.

19. $x = t$, $y = 1/(1 + t^2)$, $z = t^2$. Note that y and z are positive for all t. The curve passes through $(0, 1, 0)$ when $t = 0$. As $t \to \infty$, $(x, y, z) \to (\infty, 0, \infty)$, and as $t \to -\infty$, $(x, y, z) \to (-\infty, 0, \infty)$. So the graph is IV.

20. $x = e^{-t} \cos 10t$, $y = e^{-t} \sin 10t$, $z = e^{-t}$.
$x^2 + y^2 = e^{-2t} \cos^2 10t + e^{-2t} \sin^2 10t = e^{-2t}(\cos^2 10t + \sin^2 10t) = e^{-2t} = z^2$, so the curve lies on the cone $x^2 + y^2 = z^2$. Also, z is always positive; the graph must be I.

21. $x = \cos t$, $y = \sin t$, $z = \sin 5t$. $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder with axis the z-axis. Each of x, y and z is periodic, and at $t = 0$ and $t = 2\pi$ the curve passes through the same point, so the curve repeats itself and the graph is V.

22. $x = \cos t$, $y = \sin t$, $z = \ln t$. $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder with axis the z-axis. As $t \to 0$, $z \to -\infty$, so the graph is III.

34. The projection of the curve C of intersection onto the xy-plane is the parabola $y = x^2$, $z = 0$. Then we can choose the parameter $x = t \Rightarrow y = t^2$. Since C also lies on the surface $z = 4x^2 + y^2$, we have $z = 4x^2 + y^2 = 4t^2 + (t^2)^2$. Then parametric equations for C are $x = t$, $y = t^2$, $z = 4t^2 + t^4$, and the corresponding vector function is
$\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + (4t^2 + t^4)\,\mathbf{k}$.

38. The particles collide provided $\mathbf{r}_1(t) = \mathbf{r}_2(t) \Leftrightarrow \langle t, t^2, t^3 \rangle = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle$. Equating components gives $t = 1 + 2t$, $t^2 = 1 + 6t$, and $t^3 = 1 + 14t$. The first equation gives $t = -1$, but this does not satisfy the other equations, so the particles do not collide. For the paths to intersect, we need to find a value for t and a value for s where $\mathbf{r}_1(t) = \mathbf{r}_2(s) \Leftrightarrow \langle t, t^2, t^3 \rangle = \langle 1 + 2s, 1 + 6s, 1 + 14s \rangle$. Equating components, $t = 1 + 2s$, $t^2 = 1 + 6s$, and $t^3 = 1 + 14s$. Substituting the first equation into the second gives $(1 + 2s)^2 = 1 + 6s \Rightarrow 4s^2 - 2s = 0 \Rightarrow 2s(2s - 1) = 0 \Rightarrow s = 0$ or $s = \frac{1}{2}$. From the first equation, $s = 0 \Rightarrow t = 1$ and $s = \frac{1}{2} \Rightarrow t = 2$. Checking, we see that both pairs of values satisfy the third equation. Thus the paths intersect twice, at the point $(1, 1, 1)$ when $s = 0$ and $t = 1$, and at $(2, 4, 8)$ when $s = \frac{1}{2}$ and $t = 2$.

10.2: 20, 28

20. The vector equation for the curve is $\mathbf{r}(t) = \langle t^2 - 1, t^2 + 1, t + 1 \rangle$, so $\mathbf{r}'(t) = \langle 2t, 2t, 1 \rangle$. The point $(-1, 1, 1)$ corresponds to $t = 0$, so the tangent vector there is $\mathbf{r}'(0) = \langle 0, 0, 1 \rangle$. Thus, the tangent line is parallel to the vector $\langle 0, 0, 1 \rangle$ and parametric equations are $x = -1 + 0 \cdot t = -1$, $y = 1 + 0 \cdot t = 1$, $z = 1 + 1 \cdot t = 1 + t$.

28. To find the point of intersection, we must find the values of t and s which satisfy the following three equations simultaneously: $t = 3 - s$, $1 - t = s - 2$, $3 + t^2 = s^2$. Solving the last two equations gives $t = 1$, $s = 2$ (check these in the first equation). Thus the point of intersection is $(1, 0, 4)$. To find the angle θ of intersection, we proceed as in Exercise 27. The tangent vectors to the respective curves at $(1, 0, 4)$ are $\mathbf{r}_1'(1) = \langle 1, -1, 2 \rangle$ and $\mathbf{r}_2'(2) = \langle -1, 1, 4 \rangle$. So $\cos\theta = \frac{1}{\sqrt{6}\sqrt{18}}(-1 - 1 + 8) = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}}$ and $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 55^\circ$.

Note: In Exercise 27, the curves intersect when the value of both parameters is zero. However, as seen in this exercise, it is not necessary for the parameters to be of equal value at the point of intersection.